Solution and Analysis on Three-dimensional Mathematical Model of Particles Sedimentation in ESP

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Abstract: Solutions of the three-dimensional mathematical model on particles sedimentation in ESP were deduced, as well as the precipitation efficiency formula. Comparisons were given between one-dimensional mathematical model and two-dimensional mathematical model. According to the experimental results, the discussions on the concentration distribution were carried through. All of these can describe the movement of electrified particles in ESP more correctly, so as to supplying the corresponding reference base for the design, manufacture, operation and precipitating efficiency forecast of ESP.

Key words: electrostatic precipitators (ESP), three-dimensional mathematical model, precipitation efficiency, dust concentration distribution

1. Introduction

Coal-fired power plants are still in the dominant place in Chinese electric power industry. Electrostatic precipitators (ESP) are widely applied in the coal-fired units because of their high dust removal efficiency, simple manipulation and reliable operation. In fact, the movements of dust particles in the ESP are very complex. Cooperman thought there were three concentration grads in the dust removal area [1]: in the gas flowing direction, the upriver dust concentration was higher than the downriver one; in the horizontal direction which was vertical to the gas flow, the dust concentration was higher and higher when it was from corona plate to collection plate; in the gravitational direction, the dust concentration was increased from up to down. Because of the mix and diffusion effect of the turbulence flow, the dust particles would transfer from high concentration place to the low one, and this turbulent confusion was obvious contrary to the dust collection process.

By far, the widely applied efficiency formula of Deutsch had only taken the changing of dust concentration which following the direction of gas-flow into account, ignoring the other 2 factors, resulting the calculated result of the Deutsch formula bigger than the actual one, so the Deutsch formula can’t exactly describe the ESP dust removal efficiency. Based on the knowledge of the fore-people and the study actuality of electrostatic dust removal theory, it is necessary to establish a more objective mathematical formula including much more factors. In this paper, the solution of the three-dimensional mathematical model on particles sedimentation in ESP was given as well as the precipitating efficiency formula, and comparisons were given between the one dimensional model and two dimensional model. Further, according to relevant experimental results, the three dimensional concentration distributing were discussed and analyzed in detail. All of these can describe the movement of electrified particles in ESP more correctly, thus supply the corresponding reference base for the design, operation and precipitation efficiency forecast of the ESP.

2. Solutions of the three-dimensional mathematical models

2.1. Three-dimensional partial differential equation for particle sedimentation

The mechanism of the sedimentation of electrified particles in the electrostatic field of ESP is very complex. In order to simplify the mathematical model, several assumptions were given as follows [1]:

(1) The particles are global and the electrified amounts of particles with the same diameters are equal;
(2) Ignoring the mutual-effect between particles;
(3) Electric field intensity and gas ions concentration were equable near every particle;
(4) The gas flow velocity was uniform;
(5) There is not any interfere in the dust collection area;
(6) When the particle moved onto the collection plate, it was regarded as the particle was collected.

According to the above assumptions, the three dimensional reference frames was established in the electric field. The origin was in the center of the inlet plane, x axis was the gas-flow direction, y axis pointed to the collection plate, z axis was the gravitation direction, as shown in Figure 1. In Figure 1, 1 represent collection plate, 2 represent corona plate. In the reference frame, an element was selected which was shown in Figure 2.

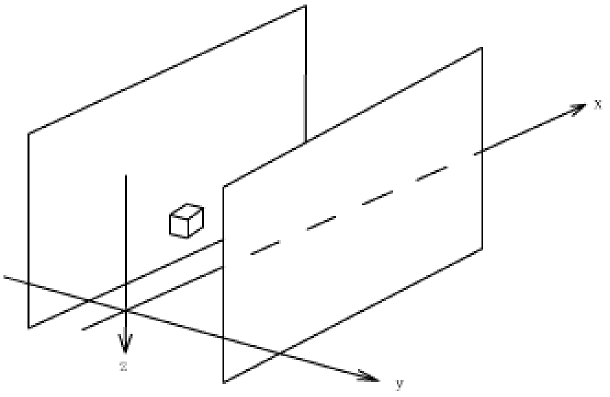

Figure 1 Sketch-map of electrostatic field in ESP

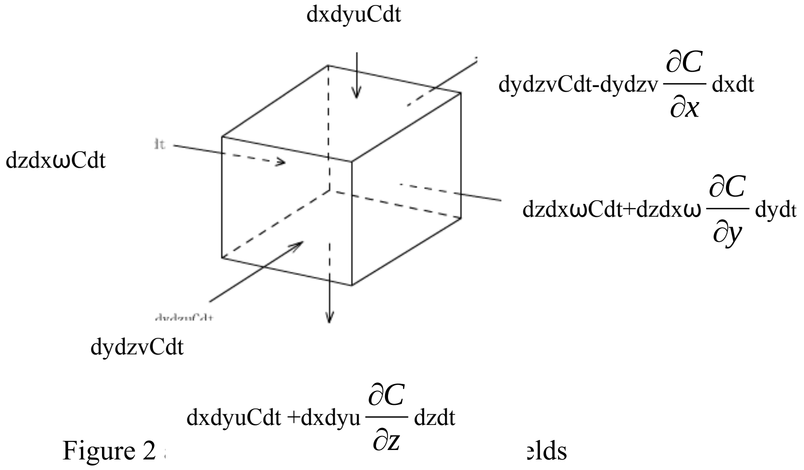

Figure 2 [illegible] elds

In the element, the dust could not been produced or eliminated, so the equation as follows can be obtained [1]

$$v\frac{\partial C}{\partial x}+\omega\frac{\partial C}{\partial y}+u\frac{\partial C}{\partial z}=0 \qquad (1)$$

C—dust concentration (mg/m^3)

V—velocity of gas-flow in x direction, it was the velocity of dust in x direction (m/s);

Ω—velocity of dust in y direction, it was the migration velocity of electrified particles (m/s);

U—velocity of dust in z direction, it was the sedimentation velocity of dust in the gravitation direction (m/s)

2.2. Solution of the one-dimensional and two-dimensional mathematical models

The given parameters were as follows: the areas of 1 m length electric field was a (m^2), the distance between corona plate and collection plate(the absolute distance between the origin and collection plate) was b (m), the length of the electric field was l (m), the section areas of ESP f □ab (m^2), the dust concentration in the inlet center was C_0 (mg/m^3).

(1) Suppose the particles concentration of some section was average, and ignore the concentration changing in the y, and z direction, equation (1) could be simplified to the following form:

$$v\frac{\partial C}{\partial x} □ 0$$

Thus, its solution can be as follows:

$$C(x) = C_0 \exp\left(-\frac{a\omega}{fv}x\right) \qquad (2)$$

The one dimensional dust removal efficiency formula (Deutsch formula) had been gotten [1]

$$\eta □ 1 □ \exp\left(-\frac{al}{fv}\omega\right)$$

Suppose the dust removal area was A,

A= al and the gas flow rate was Q, Q= fu , then:

$$\eta\square 1\square\exp(-\frac{A}{Q}\omega) \qquad (3)$$

(2) If ignoring the concentration changing in z direction, model (1) could be simplified to the following form

$$v\frac{\partial C}{\partial x}+\omega\frac{\partial C}{\partial y}\square 0$$

Then, its solution can be obtained as follows:

$$C(x, y) = C_0 \exp[(-\frac{a\omega}{fv}x+\frac{a}{f}y) \qquad (4)$$

So, the two dimensional dust removal efficiency formulas could be achieved as the following form:

$$\eta\square 1\square 1.718\exp(-\frac{al}{fv}\omega)$$

$$\square 1\square 1.718\exp(-\frac{A}{Q}\omega) \qquad (5)$$

2.3. Solution of three-dimensional model of the particles sedimentation [2-3]

According to the above particle-sedimentation three-dimensional mathematical model, f=C(x, y, z) was a curve whose vector was $\square\frac{\partial C}{\partial x}\square\frac{\partial C}{\partial y}\square\frac{\partial C}{\partial z}\square$and based on formula (2), (3) the form of the equation was

$$C(x, y, z)\square C_0 e^{k_1x+k_2y+k_3z} \qquad (6)$$

Where: k_1--concentration distribution coefficient in x direction; k_2--concentration distribution coefficient in y direction; k_3—concentration distribution coefficient in z direction.

In the reference frame of Figure 1, the boundary conditions were:

(1) $C(0, 0, 0) = C_0$

(2) $C(0, y, z)=C_0 e^{k_2y+k_3z}$ (7)

Where: C_0 was the dust concentration of origin. The height of ESP were thought relative infinite, so f=C(x, y, z) could be solved according to the above boundary condition.

By Introducing (6) into (1), it will change into:

$$C_0 e^{k_1x+k_2y+k_3z}(vk_1+\omega k_2+uk_3)=0$$

Because $C_0 e^{k_1x+k_2y+k_3z}>0$,

$vk_1+\omega k_2+uk_3=0$, and $k_1=-\frac{\omega k_2+uk_3}{v}$,

$$C(x, y, z) = C_0\exp[(-\frac{\omega k_2+uk_3}{v})x+k_2y+k_3z]$$

The average dust concentration in the out section of ESP (while x=l, the electric height was a/2) was

$$\overline{C}\square\frac{C_0}{ab/2}\int_0^b\int_{-a/4}^{a/4}C(l,y,z)dydz$$

$$\square\frac{C_0}{ab/2}\cdot$$

$$\text{Exp}[(-\frac{\omega k_2+uk_3}{v})l]\int_0^b\int_{-a/4}^{a/4}e^{k_2y+k_3z}$$

$$\square\frac{C_0}{k_2k_3ab/2}[e^{k_2b}\square 1][e^{k_3a/4}\square e^{-k_3a/4}]\cdot$$

$$\exp\square-\frac{uk_3l}{v}\square\exp[\frac{f}{a}k_2(-\frac{al}{fv}\omega)]$$

In the equation (4):

$C(x, y)=C_0\exp[(-\frac{a\omega}{fv}x+\frac{a}{f}y)$, so $k_2\square\frac{a}{f}$,

and $\overline{C}\square\frac{C_0}{ak_3/2}[e\square 1][e^{k_3a/4}\square e^{-k_3a/4}]\cdot$

$$\exp\square-\frac{uk_3l}{v}\square\exp(-\frac{A}{Q}\omega)$$

The dust removal efficiency could be gotten

$$\eta\square 1-\frac{\overline{C}}{C_0}$$

$$\square 1-\frac{1}{ak_3/2}[e\square 1][e^{k_3a/4}\square e^{-k_3a/4}]\cdot$$

$$\exp\square-\frac{uk_3l}{v}\square\exp(-\frac{A}{Q}\omega)$$

$$\square 1\square 1.718\frac{e^{k_3a/4}-e^{-k_3a/4}}{ak_3/2}\cdot$$

$$\exp\square-\frac{uk_3l}{v}\square\exp(-\frac{A}{Q}\omega) \qquad (8)$$

Suppose that:

$$K_h\square 1.718\frac{e^{k_3a/4}-e^{-k_3a/4}}{ak_3/2}\cdot\exp\square-\frac{uk_3l}{v}\square$$

$$\square 1.718\times 4\frac{\mathrm{sh}(k_3a/4)}{ak_3}\cdot\exp\square-\frac{uk_3l}{v}\square \qquad (9)$$

$$\eta\square 1\square K_h\exp(-\frac{A}{Q}\omega) \qquad (10)$$

In the equation (9), when k_3—>0(ignoring the concentration distributing in z direction), K_h=1.718. Thus, equation (10) was the same as equation (5), i.e. equation (5) was only the special case of model (10). If k_3 were confirmed, classification dust removal efficiency could be obtained from model (10), but it was very difficult.

3. Analysis and Discussions

In this paper, k_3 was decided based on the experimental data which were carried out on the specially established ESP test equipment. The concrete dimensions were as follows: plate height h=0.2m, plate length l=0.55m, the distance between corona plate and collection plate b=0.05m, working voltage V=30kV. Based on the above parameters, the particles migration velocity was:

$$\omega□\frac{\varepsilon_P}{\varepsilon_P+2}\frac{\varepsilon_0 E_0 E_P}{\mu} d_P$$

$$□11.8\times10^4 d_P□m/s□ \qquad (11)$$

Where, ε_P —relative permittivity, equal to 4;

ε_0 —vacuum permittivity, its value was 8.85×10^{-12} ($C^2/N\cdot m^2$);

E_0—electrified area intensity, let $E_0=E_P$□V/m□;

E_P—electric field intensity near dust removal area, its value was 6×10^5(V/m).

μ—glutinosity coefficient, its value was $1.8\times10^{□5}$(Kg/m·s);

When the gas flow velocity in the dust removal area was 1.0m/s, the different classification dust removal efficiencies including the testing efficiency, the efficiency calculated with Deutsch efficiency formula and the efficiency calculated with two dimensional formula (5) were shown in Figure 3 and Figure 4. In fact, Figure 4 was the part magnification of Fig3. When the particles diameter was bigger than 10μm, the efficiencies calculated by Deutsch efficiency formula, two dimensional efficiency formula, and three dimensional efficiency formula were in closely agreement. Therefore, we just compared and analyzed the particles whose particle diameters were less than 10μm.

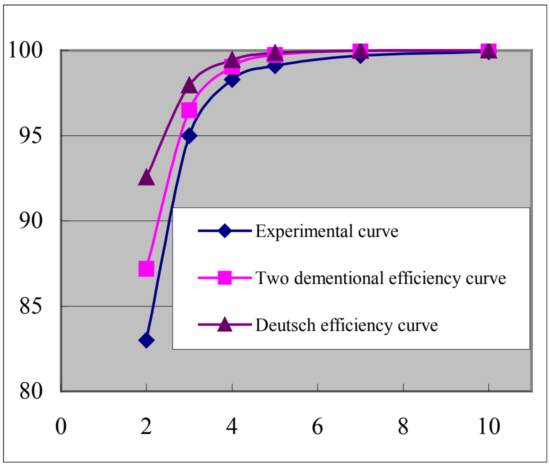

Figure 3 Comparison on classified efficiency

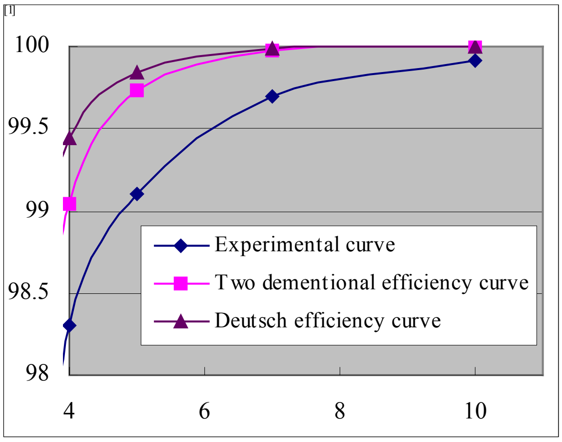

Figure 4 Zoom out on part of Figure 3

Based on the real classification efficiencies in Figure 3 and Figure 4, and according to the formula (8), (9), and (10), we can obtain different ω□k_3, K_h with different particles, which were shown in Table 1.

Table 1 Calculated results of k_3 and K_h

Diameter d_p□μm□	Migration velocity ω□m/s□	k_3	K_h
2	0.236	13.43	2.279
3	0.345	15.26	2.4549
4	0.472	19.86	3.0569
5	0.59	30.9	5.9277
7	0.826	51.63	26.4946
10	1.18	84.43	346.72

As can be seen from Table 1, we know that ω□k_3□K_h, and the particles diameters are proportional related; K_h>1.718; K_3 was a constant, and it was a concentration distribution coefficient affected by comprehensive factors.

In order to simplify calculations, K_h was regarded as a constant. When it was 2.279, 2.4549, 3.0569, and 5.9277, the calculated classification dust removal efficiencies and their compares with two dimensional dust removal efficiencies, Deutsch dust removal efficiency and real classification dust removal efficiencies were shown in Table 2 and Figure 5. As can be seen, we know that when K_h was in the range of 2~3, the calculated classification dust removal efficiencies were closely consistent with the reality.

Table 2 Comparison of dust removal efficiency

Diameter□μm□	2	3	4	5	7	10
Efficiency□K_h=2.279,%□	83.01	95.36	98.73	99.65	99.97	99.999
Efficiency□K_h=2.4549,%□	81.69	95	98.63	99.63	99.97	99.999
Efficiency□K_h=3.0569,%□	77.204	93.775	98.3	99.536	99.965	99.999
Efficiency□K_h=5.9277,%□	55.796	87.929	96.7	99.1	99.93	99.999
Two-dimensional dust removal efficiency□%□	87.186	96.5	99.044	99.739	99.981	99.9996
Deutsch dust removal efficiency□%□	92.543	97.964	99.444	99.848	99.989	99.9998
Experimental efficiency□%□	83.0	95.0	98.3	99.1	99.7	99.92

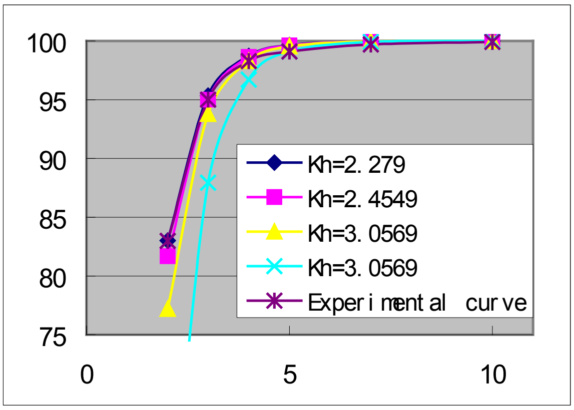

Figure 5 Comparison of efficiency

4. Conclusions

By resolving the three-dimensional mathematical model of particles sedimentation in ESP, the corresponding dust removal efficiency had been given.

Through real classification dust removal efficiency, the concentration distribution coefficient in z direction was confirmed.

K_h>1.718. When K_h was in the range of 2~3, the calculated classification dust removal efficiencies were very closely consistent with the reality.

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